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N<sup>o</sup>. XX.

*Dr. Rittenhouse, to Mr. Patterfon, relative to a method of finding the sum of the several powers of the Sines, &c.*

DEAR SIR,

Read May 18, 1792. **I** Had discovered a very elegant theorem for determining the times of vibration of a pendulum in given arches of a circle; but it included a problem the solution of which I do not remember to have met with, though I cannot suppose that it has escaped the notice of mathematicians. It is, to find the sums of the several powers of the sines, either to a radius of unity or any other.

I was induced to attempt the means of doing this solely by its usefulness, but in prosecuting the enquiry I found much of that pleasing regularity, the discovery of which the geometrician often thinks a sufficient reward for his labours.

The sums of the odd powers of the sines bear a very simple relation to each other, and so do the sums of the even powers. But all the sums of the odd powers are incommensurable to all those of the even powers.

If we take the radius equal to unity the sum of all the sines, or their first powers, will be = 1, and the sum of all their squares =  $\frac{1}{2}$  multiplied by the arch of  $90^\circ$ . The sum of all their cubes is =  $\frac{2}{3}$ , and the sum of their fourth powers =  $\frac{3}{8}$  multiplied by the arch of  $90^\circ$ . The sum of the fifth powers is =  $\frac{8}{15}$ , and the sum of the 6th powers =  $\frac{5}{16} \times$  by the arch of  $90^\circ$ .

I have not been able strictly to demonstrate any more than the two first cases. The others were investigated by the method of infinite series so far as to leave no doubt of

the ultimate ratio which the sum of the given power of the sines bears to a known power of the radius.

Having proceeded so far as the 6th power the law of continuation became evident; so that, should any problem in mathematical philosophy require it, we may proceed as far as we please in summing the powers of the sines. The law is this,

Make a fraction whose denominator is the index of the given power, and its numerator the same index, diminished by unity, and multiplied by the square of the radius; by this fraction multiply the sum of the next but one lower power, and we have the sum of the given power.

Thus 1st, the sum of the 1st power of the sines	}	By Demonstration.
is = $rr$ , or the square of the radius		
2d, sum of the 2d, power or squares is	}	By Infinite Series.
= $\frac{1}{2}rr \times$ by the arch of $90^\circ$ .		
3d, sum of the 3d, power or cubes is	}	By Infinite Series.
$\frac{2}{3}rr$ of the 1st, or = $\frac{2}{3}r^4$		
4th, sum of 4th powers is = $\frac{3}{4}rr$ of the 2d	}	By the Law of Continuation.
or = $\frac{3}{8}r^4 \times$ by the arch of $90^\circ$ .		
5th, sum of 5th. powers is = $\frac{4}{5}rr$ of the 3d,	}	By the Law of Continuation.
or = $\frac{8}{15}r^6$ .		
6th, sum of 6th. powers is = $\frac{5}{6}rr$ of the 4th	}	By the Law of Continuation.
or = $\frac{5}{16}r^6 \times$ by arch of $90^\circ$ .		
7th, sum of 7th. powers is = $\frac{6}{7}rr$ of the 5th,	}	By the Law of Continuation.
or = $\frac{1}{3} \frac{6}{5} r^8$ .		
8th, sum of 8th. powers is = $\frac{7}{8}rr$ of the 6th,	}	By the Law of Continuation.
or = $\frac{1}{1} \frac{3}{2} \frac{5}{8} r^3 \times$ by the arch of $90^\circ$ .		
&c. &c.		

Should your leisure permit you to give any attention to this subject I shall be glad to see you furnish a demonstration for the 3d, or any subsequent case abovementioned.

I am, Sir,

Your most obedient humble servant,  
DAVID RITTENHOUSE.

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